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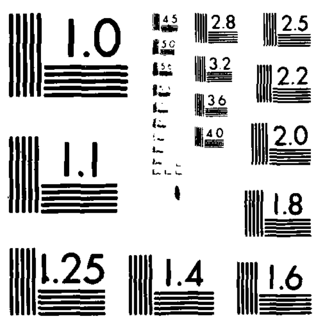
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This report presents a method combining simulation and optimization techniques to determine guidelines for operating selective withdrawal reservoir structures to meet downstream water temperature objectives. Optimal operation is achieved when operating rules that anticipate future critical temperature conditions are successfully applied.

In this study, a one-dimensional reservoir thermal simulation model

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developed by the U. S. Army Engineer Waterways Experiment Station was used to simulate the thermal stratification cycle of a reservoir. The model was interfaced with a formulation called objective-space dynamic programming (OSDP) to develop the optimal operation strategy for each decision period. The OSDP formulation retains the integrity of the simulation model and minimizes an objective function related to deviations of predicted release temperature from downstream target temperature over a portion of the stratification cycle. Application to a case study shows the potential for using the dynamic programming technique, as compared to the normal period-by-period operation, to improve performance of the system.

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PREFACE

Development of the procedures reported herein was sponsored by the Office, Chief of Engineers, as part of the Environmental and Water Quality Operational Studies (EWQOS) Program, under Task II.D, Reservoir Regulation for Water Quality Management. The investigation was conducted and the report prepared during the period June 1980 to May 1981 by Dr. Darrell G. Fontane and Dr. John W. Labadie of Colorado State University and Mr. Bruce Loftis of the Waterways Experiment Station (WES), Hydraulics Laboratory. Mr. H. B. Simmons, Chief of the Hydraulics Laboratory, WES; Dr. John Harrison, Chief of the Environmental Laboratory, WES; Mr. Frank A. Herrmann, Assistant Chief of the Hydraulics Laboratory; Mr. John L. Grace, Jr., Chief of the Hydraulics Structures Division; Dr. Dennis R. Smith, Chief of the Reservoir Water Quality Branch; and Dr. Jerome L. Mahloch, Program Manager of EWQOS, provided general guidance.

Director of WES during the investigation was COL N. P. Conover, CE. Technical Director was Mr. F. R. Brown.

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CONTENTS

Page

PREFACE 1

PART I: INTRODUCTION 3

Objective 3
Reservoir Regulation 3

PART II: SOLUTION STRATEGIES 8

Static-Optimal Release Strategy 8
Dynamic-Optimal Release Strategy 9

PART III: DYNAMIC PROGRAMMING NOTATION 11

PART IV: PREVIOUS APPROACHES 13

PART V: OBJECTIVE-SPACE DYNAMIC PROGRAMMING 17

PART VI: COMPUTATIONAL EXPERIENCE 27

PART VII: CONCLUSIONS AND EXTENSIONS 31

REFERENCES 33

OPTIMAL CONTROL OF RESERVOIR DISCHARGE QUALITY THROUGH SELECTIVE WITHDRAWAL

Hydraulic Laboratory Investigation

PART I: INTRODUCTION

1. Federal agencies involved in the operation of systems of man-made lakes are confronted with a complexity of interdependent decisions within a multipurpose framework. Federal reservoir projects are undertaken for such purposes as flood control, navigation, hydropower, water supply, recreation, and water quality control. Of these, flood control and water quality control present the greatest challenges for effective operation. In management for these purposes, rapid decisions are often required, decisions that must be made with uncertainty about future conditions and that can have a significant impact on other project purposes.

Objective

2. The purpose of this report is to discuss in general the management of density-stratified reservoirs for water quality control and to present a procedure for determining optimal release strategies to meet downstream temperature requirements. The procedure is based upon a combined simulation-optimization approach in which a reservoir water quality simulation model is used to evaluate alternatives suggested by an optimization method. Temperature is the only water quality parameter considered in the procedure. The procedure can be extended to include additional water quality parameters, but it is difficult to develop an objective function which accurately reflects preferences between parameters that sometimes conflict.

Reservoir Regulation

3. Many reservoirs can be operated to release water of a specific

quality. In general, the quality of the water within a reservoir varies with both time and space. The variation is usually most pronounced during thermal stratification, when surface water is warmed and cooler water remains near the bottom (Figure 1). The resulting density-stratified condition inhibits vertical mixing and affects various

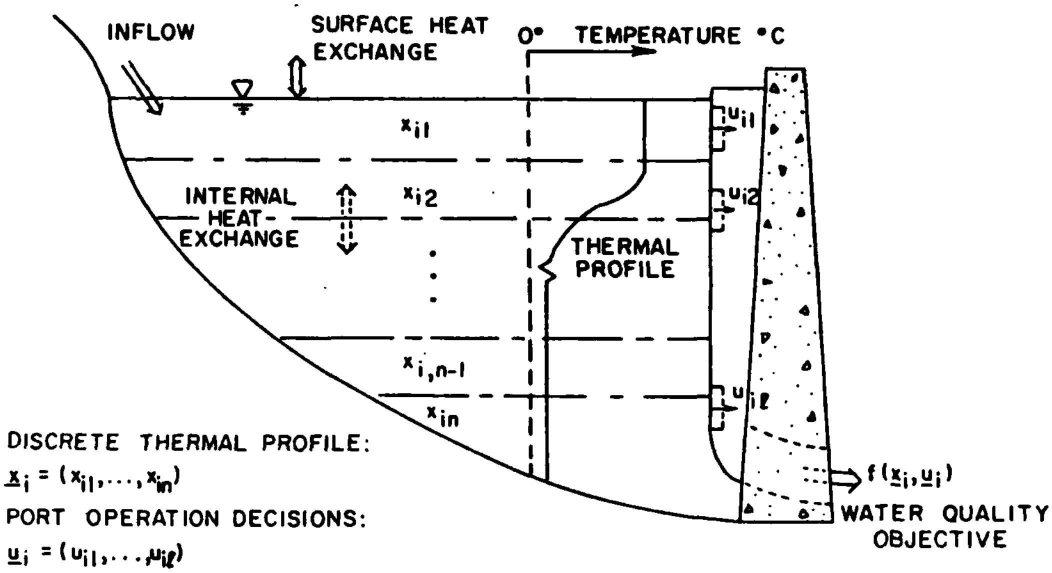


Figure 1. Representation of selective withdrawal structure operation for a thermally stratified reservoir

hydrodynamic processes within the reservoir. As a consequence, the quality of the water varies with its location in the reservoir. Furthermore, this variation is generally most pronounced vertically. With knowledge of the vertical distribution of temperature within a reservoir, a selective withdrawal outlet works (Figure 2) which provides the flexibility of withdrawing water of the desired quality from various strata in the lake can be designed.

4. Reservoirs can be operated to achieve in-lake objectives such as evacuating waters with low dissolved oxygen content from the bottom

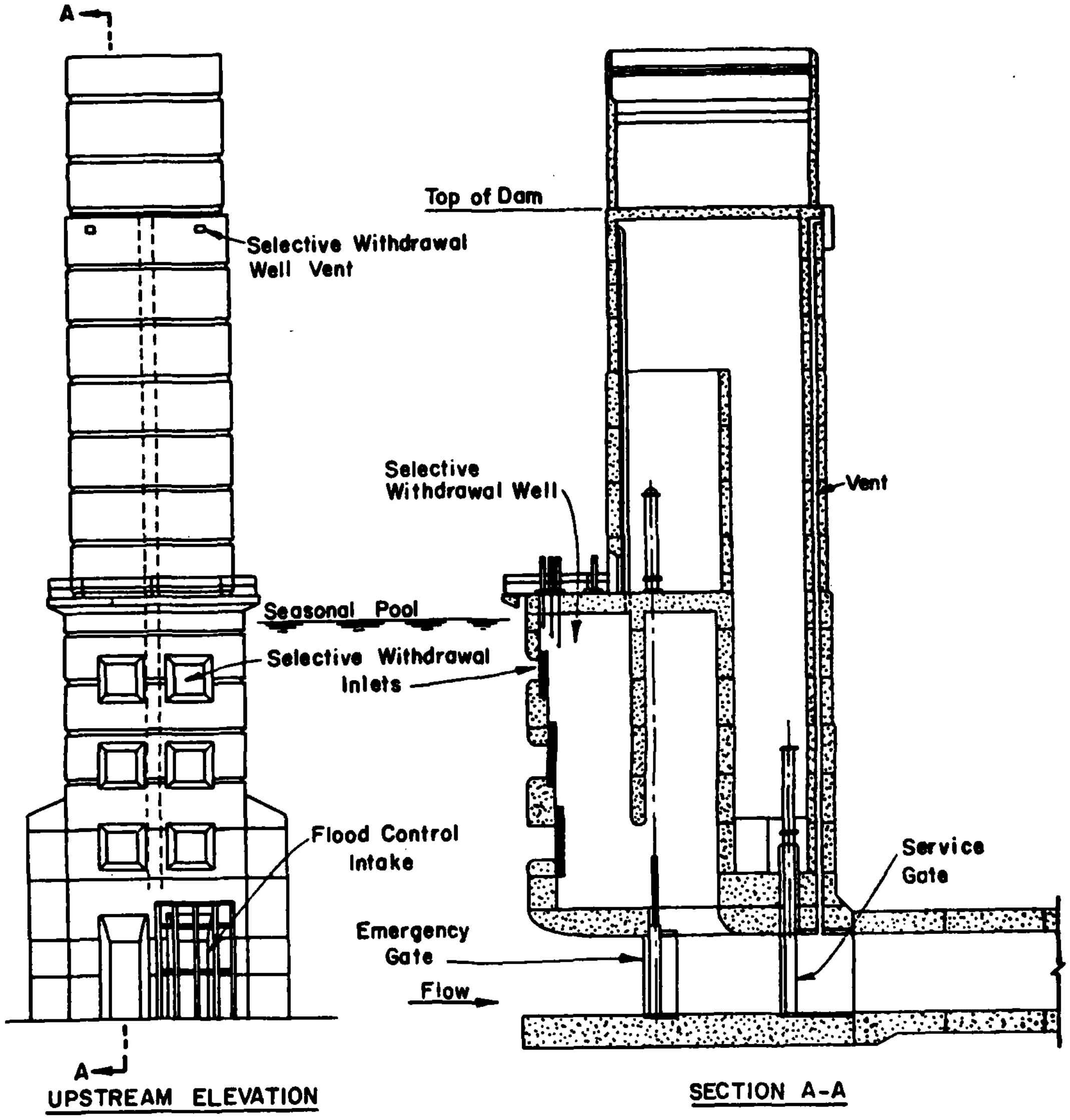


Figure 2. Example of selective withdrawal structure

or releasing a density current of inflowing suspended sediment resulting from a storm in the upstream watershed. Most often, however, reservoirs are operated to meet downstream temperature objectives during the thermal stratification cycle. Downstream temperature objectives are established to enhance a coldwater or warmwater fishery or to maintain pre-project stream temperature conditions. Release temperature can also be

important for other reasons such as irrigation.

5. Operational decisions for daily control of release temperature are usually based on current conditions. Thermal simulation models are used to evaluate the capability of selective withdrawal designs to meet release temperature objectives by using hydrologic and meteorologic data to simulate the thermal stratification cycle. Algorithms are included within the models to simulate operation of the selective withdrawal structure. Since the algorithms used incorporate only current and not anticipated future conditions, any sequential operational decisions must be regarded as myopic. From consideration of in-lake temperature conditions, flow requirement, outlet works geometry, and downstream target temperature, daily decisions can be made as to which selective withdrawal ports should be open and what flow should be released through each outlet.

6. If the withdrawal structure is adequately designed and if the reservoir is large enough that sufficient water of the desired temperature is available to ensure meeting temperature objectives over the stratification cycle, then this period-by-period or myopic operation is satisfactory. However, if the reservoir can provide only a limited quantity of water at a desired thermal level, a severe deviation from the temperature objectives may occur during the latter part of the cycle (Figure 3). The release temperature may be undesirable for the water's intended use. Reducing the severity of the deviations from the downstream target temperature over a longer period of time will reduce the shock effect of abrupt temperature changes on the river ecosystem. In the case of limited capacity, the way to produce desired release temperatures over the entire stratification cycle may be to minimize: (a) the maximum deviation of the release temperature from the target level, (b) the sum of the absolute deviations (or squared deviations), or (c) the weighted sum of the absolute deviations during critical periods such as fish spawning.

7. In this study, a thermal simulation model was used to evaluate the ability of selective withdrawal structure designs to meet release

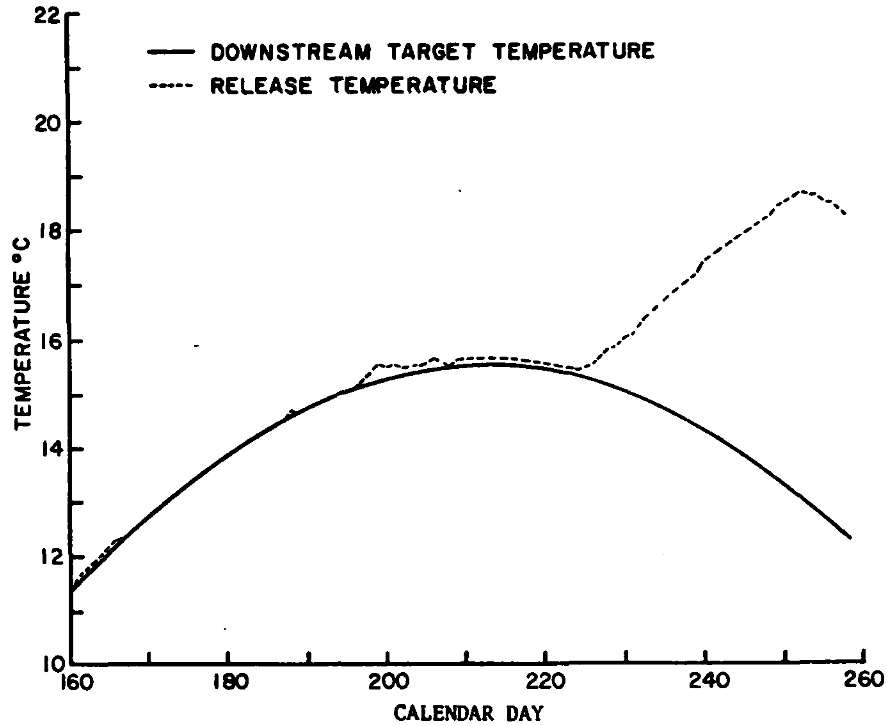


Figure 3. Comparison of release temperature and target temperature for a case study reservoir

temperature objectives at proposed reservoirs. The mathematical model incorporates hydrologic and meteorologic data to simulate the thermal stratification cycle in a reservoir and includes algorithms to simulate operation of each selective withdrawal structure.

PART II: SOLUTION STRATEGIES

Static-Optimal Release Strategy

8. The reservoir operation release strategy selected for water quality control is often identified on a daily basis, and the decisions involved are made based only on the current state of the system; no consideration is given to future conditions. Such decisions can be regarded as myopic. In following such a strategy, if decisions are made such that the release temperature corresponds most nearly to a downstream temperature objective, then the strategy can be called static-optimal. Determination of a static-optimal release strategy is not a trivial exercise. The decisions to be made involve the operation of a multilevel outlet structure (such as shown in Figure 2) to release water with a temperature that most nearly matches downstream temperature requirements. Specifically, the decisions are which of the selective withdrawal ports should be opened, whether the larger floodgate should be opened, and what flow should be released through each of the open outlets. Static-optimal decisions include the following considerations:

- *a*. State of the system; that is, the vertical temperature profile in the lake on the day of interest.
- *b*. Morphological description of the lake. Because the volume varies with depth, the volume of water available for release at a specified temperature is a function of its vertical location in the lake.
- *c*. Total flow to be released downstream. Usually this is specified by some other project purpose such as hydropower or flood control.
- *d*. Downstream target temperature.
- *e*. Hydraulic constraints of the outlet structure. Each of the outlets shown in Figure 2 is limited by a minimum flow rate, or rate below which flow cannot be controlled, and a maximum flow rate. Also, the entire selective withdrawal system has a maximum design capacity. And each different outlet configuration can present different constraints. Sometimes for hydraulic reasons two particular ports cannot be operated simultaneously; often it is preferable to operate vertically adjacent ports.

The objective function for the static-optimal reservoir regulation problem is to minimize the difference between the release temperature and the downstream target temperature based on information available at the time of interest. If there is sufficient volume available in the lake and sufficient flexibility provided in the design of the outlet structure, then a static-optimal release strategy will result in release temperatures which deviate only slightly from the downstream target temperatures for the entire stratification cycle.

Dynamic-Optimal Release Strategy

9. For many projects, a static-optimal release strategy is entirely acceptable. However, there are projects for which it is important to minimize the deviations or to redistribute them in time. A dynamic-optimal release strategy requires decisions based on anticipated future conditions. Whereas the static-optimal objective is to minimize the deviations for each day of operation or simulation, the dynamic-optimal objective is to minimize some function of all of the temperature deviations for all of the days in the simulation period. The static-optimal problem can be expressed mathematically as

$$\min_{\underline{\phi}^n} (T_r - T_t)^2 \quad \text{for every } n \tag{1}$$

subject to: simulation model

where

T_r = predicted release temperature

T_t = downstream temperature objective or target temperature

$\underline{\phi}^n$ = decision space; that is, the combination of ports that should be opened and the associated flow rates of each for day n

A typical dynamic-optimal problem can be expressed mathematically as

$$\min_{\underline{\phi}} \left[\sum_n (T_r - T_t)^2 \right] \tag{2}$$

subject to: simulation model

where ϕ now represents a decision matrix for every time step of the simulation period. Typical solutions to these two problems are shown in Figure 4.

10. The dynamic-optimal release strategy can be based on any of several objectives such as minimizing the sum of the squares of the deviations, as above; minimizing the sum of the absolute deviations throughout the simulation period; minimizing the maximum deviation; or satisfying some smoothness criterion for the daily deviations. The objective could even be maximizing the sum of absolute deviations to determine the worst possible operational strategy an esulting downstream temperatures. Such information could provide a better overall understanding of the reservoir system under consideration. The primary purpose of this report is to present a procedure for the solution of the dynamic-optimal reservoir regulation problem.

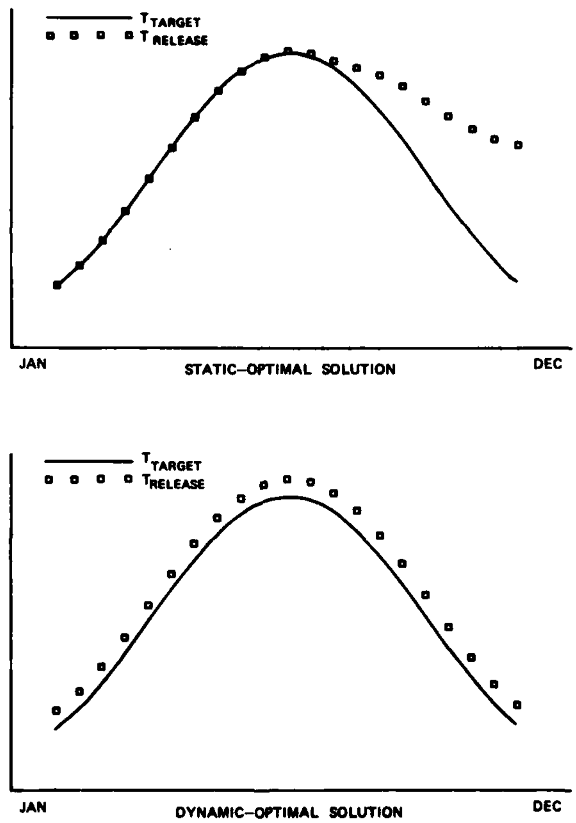
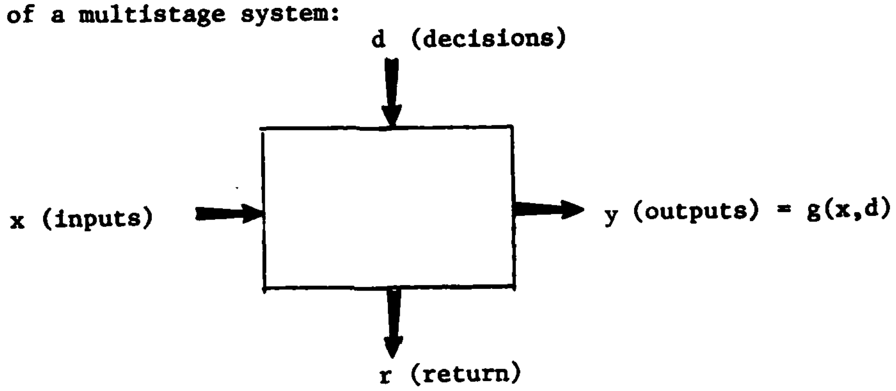


Figure 4. Typical system response to optimal release strategies

PART III: DYNAMIC PROGRAMMING NOTATION

11. Optimization means finding the best solution among several feasible alternatives. Dynamic programming is an approach to optimization that takes a sequential or multistage decision process containing many interdependent variables and reduces it to a series of single-stage problems, each containing only a few variables. Reservoir operation is a sequential decision process conducive to solutions by dynamic programming.

12. Nemhauser (1966) has used five variables to characterize one stage of a multistage system:



a. An input state x that provides all relevant information about inputs to the stage.

b. An output state y that provides all relevant information about outputs from the stage.

c. A decision variable d that controls the operation of the stage.

d. A stage return r that is a scalar variable which measures the utility of the decision.

e. A stage transformation g that is a single-valued transformation whic'` expresses each component of the output state as a function of the input state and the associated decisions; that is, $y = g(x,d)$.

The dynamic-optimal reservoir regulation problem can be formulated as a resource allocation problem; that is, best allocation of a limited resource to each of many stages. Using the notation of Nemhauser, the forward recursive equation for a general stage i for a simple resource allocation problem can be written as

$$f_i(x_i) = \min_{d_i} \left[r(x_i, d_i) + f_{i-1}(x_{i-1}) \right] \tag{3}$$

$$\text{subject to:} \quad x_i = x_{i-1} + d_i \tag{4}$$

for $i = 1, 2, \ldots, N$

where

- i = stage index
- x = state variable; the quantity of the resource allocated up to and including the current stage
- d = decision variable; the quantity of the resource allocated at the current stage
- r = contribution to the objective function at the current stage
- f = accumulated contributions to the objective function, assuming the previous decisions were optimal
- N = total number of stages

This recursive relation will serve as a reference for notation and concepts as objective-space dynamic programming is presented and its application to reservoir regulation for water quality management is discussed in the remainder of this report.

PART IV: PREVIOUS APPROACHES

13. The use of simulation models in conjunction with optimization methods to determine reservoir operation strategies to satisfy water quality objectives is a relatively new procedure in the field of water resources systems analysis. The procedure has been used more often for planning and less often for actual operation. For example, it has been used to determine sizes and locations of selective withdrawal intakes (Loftis and Fontane 1976) and to develop improved or simplified operational techniques (Patterson et al. 1977; Maynord et al. 1978). Beard and Willey (1970) developed a thermal simulation model that includes a heuristic procedure to anticipate future temperature objectives in determining reservoir operational strategies. Kaplan (1974) combined a reservoir ecosystem simulation model and a nonlinear optimization technique to determine the best mode of operation of a selective withdrawal outlet structure considering constraints of various water quality parameters. A scalar index that commensurates and prioritizes several water quality objectives was used as the objective function for this optimization problem. Kaplan's model solves the static-optimal release problem and thus does not anticipate future conditions.

14. In research performed for the Waterways Experiment Station at Colorado State University, Farber (1978) combined a state-space dynamic programming algorithm with the WESTEX Reservoir Heat Budget Model (Loftis 1979). This combination provided a systematic procedure for determining release temperature regulation strategies that anticipated future meteorological and hydrological conditions. Dynamic programming was selected because it could handle sequential decisions and system nonlinearities conveniently.

15. Farber formulated the dynamic programming problem by representing the state of the system $\underline{x}_i$ for simulation period i as the vector of temperatures corresponding to the various discretized vertical layers. The decisions $\underline{u}_i$ for period i were the port selection decisions; i.e., which ports should be open and what flow should be released through eac , open port. The return function for simulation

period i was the squared difference between the downstream target temperature T_t, and the predicted release temperature T_r, or

$$r_i(\underline{x}_i,\underline{u}_i) = (T_t - T_r)^2 \tag{5}$$

The objective function was then the sum of squared deviations for all simulation periods. The state transformation vector function

$$\underline{x}_{i+1} = g_i(\underline{x}_i,\underline{u}_i) \tag{6}$$

which is part of the WESTEX simulation model, provided the state of the system at the next time step based on the current state and current decisions.

16. Farber used a proposed reservoir in the Southeast as a case study and demonstrated the capability of the combined WESTEX-dynamic programming approach to "save" cold water during an early simulation period in anticipation of later needs. It worked better than the myopic static-optimal approach, but the small size of the case study reservoir limited the volume of cold water which could be retained for later use.

17. Farber's use of the combined simulation-dynamic programming model involved some computational difficulties. The state vector could easily have had a dimension of 30 or more, depending upon the number of vertical layer discretizations necessary to represent the temperature profile. In order to deal with the dimensionality problem, the temperature profile vector $\underline{x}_i$ was represented by third-order Chebyshev polynomials. The coefficients of the polynomials became pseudo-state variables because they, rather than the actual state vector (temperature profile), were manipulated. If a higher order polynomial is needed to adequately describe the temperature profile, then the problem can become intractable. Although representing the temperature profile by orthogonal polynomials reduced the dimensionality of the problem, the problem remained severe because time-consuming simulation was used directly in the computation of the dynamic programming objective function.

18. Recently, new ways of formulating dynamic programming problems have been developed to deal with the so-called "curse of dimensionality." Conventional dynamic programming approaches are state-space approaches; that is, the dynamic programming optimal value or return function is defined over the space or range of the state variables of the system. Labadie and Hampton (1979) developed a dynamic programming formulation for problems where the decision space is much more restricted than the state space. An example of this condition would be a reservoir operational problem where the maximum controlled release (the decision variable) is much smaller than the active storage volume in the reservoir (the state variable). The decision-space approach basically solves the dynamic programming problem by developing a surrogate state variable which represents the accumulated decisions from stage 1 through stage i. The true state of the system at stage i is reconstructed from the initial true state at stage 1 and the accumulation of decisions from stage 1 through stage i. Thus, there may be easier ways of solving the problem by using certain surrogate state variables of lower dimension, rather than the actual, high-dimensional state vector. Bertsekas (1976) relates this approach to the concept of a sufficient statistic; that is, identification of the minimum amount of information needed at any given stage of a sequential decision process such that subsequent decisions based on that information will be optimal or at least suboptimal.

19. Tauxe et al. (1980) developed an important extension of dynamic programming, called multiobjective dynamic programming, to solve problems involving multiple noncommensurate objectives. The procedure treats all objectives other than the primary one as constraints. Each objective considered as a constraint is added as a state variable to the dynamic programming formulation. The multiobjective problem is redefined as a single-objective problem with multiple state variables. The solution to this problem yields the value of the primary objective as a function of the remaining objectives. The dynamic programming optimal value or return function must be evaluated over all original state variables, plus the additional state variables. Though methods like

discrete differential dynamic programming can be applied in this case, the computational burden can be great for problems with several state variables and/or several objectives.

PART V: OBJECTIVE-SPACE DYNAMIC PROGRAMMING

20. The problem of determining optimal operational strategies for selective withdrawal structures, originally solved by Farber, can be solved over an objective space without the need to include the original state variables in the dynamic programming (DP) optimal value function. This objective-space dynamic programming (OSDP) approach is extremely powerful since the original multidimensional problem can be reduced to a one-dimensional dynamic programming problem. The OSDP concept will now be explained in the context of the problem of selective withdrawal structure operation.

21. Suppose, for instance, that an operational policy for a selective withdrawal structure is desired such that the sum of the squared deviations of the release temperature T_r from the target temperature T_t is minimized over L time periods. That is, the goal is to minimize S, where

$$S = \sum_{i=1}^{L} (T_{r,i} - T_{t,i})^2 \tag{7}$$

Let the squared deviation for stage i be

$$\Delta D_i = (T_{r,i} - T_{t,i})^2 \tag{8}$$

Then, the sum of the squared deviations is

$$S = \sum_{i=1}^{L} \Delta D_i \tag{9}$$

Define D_i as the accumulated values of the squared deviations from stage 1 through stage i

$$D_i = D_{i-1} + \Delta D_i \ , \quad i = 1, \ldots, L \qquad (10)$$

where D_0 equals zero. Hypothetically, plots of D_i and ΔD_i versus stage i would have the form shown in Figure 5. The objective function can now be simply written as: minimize D_L .

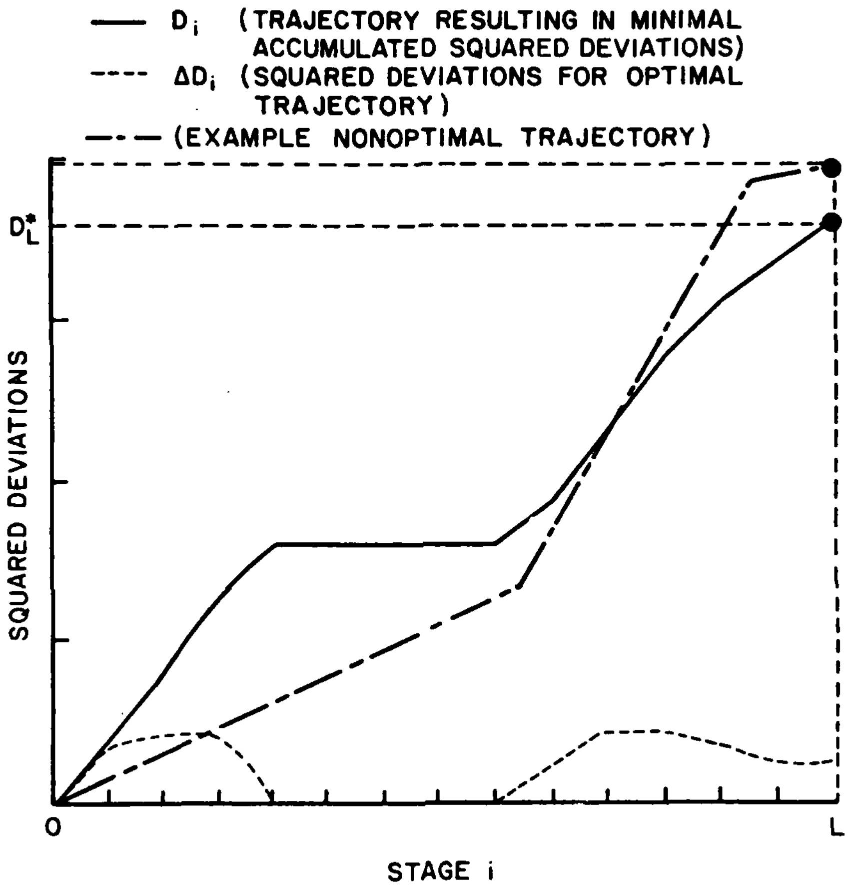


Figure 5. Hypothetical plots of D_i and ΔD_i versus stage i

22. Notice that the plot of D_i , as generated by a particular operational strategy, gives the appearance of a state trajectory. If it can be assumed that the optimal values of the squared deviations and accumulated squared deviations, ΔD_i^* and D_i^* , respectively, are generated by a <u>unique</u> operational strategy, then this optimal strategy can be determined from the values of the squared deviations at each stage ΔD_i^*

and the initial state of the system by solving an inverse problem. Under this uniqueness assumption, if the ΔD_i^* values are known, it is possible to reconstruct the unique operational strategy that produced them. Assuming the initial temperature of the reservoir is known, a forward-looking DP procedure can be used to find these ΔD_i^* values. The reason for selecting the forward DP instead of the more common backward DP procedure will be apparent from the following discussion.

23. The forward DP problem proceeds as follows. For stage 1, the initial state of the reservoir (the discrete initial temperature profile $\underline{x}_0$) is known and a set of discrete D_1 values is selected. From Equation 10, values of ΔD_1 which correspond to the values of D_1 are found. Note that the intent is to release water from the reservoir with release temperature $T_{r,1}$ such that Equation 8 is satisfied:

$$\Delta D_1 = (T_{r,i} - T_{t,1})^2 \tag{11}$$

To accomplish this, a modified target temperature $\bar{T}_{t,1}$ is specified that deviates from the actual target temperature by $(\Delta D_1)^{1/2}$; that is,

$$\bar{T}_{t,1} = T_{t,1} + (\Delta D_1)^{1/2} \tag{12}$$

Notice that $\bar{T}_{t,1} \geq T_{t,1}$ can be assumed for this case since slightly warmer water should be released if a coldwater objective is being maintained, in order to avoid severe shortages of cold water later. For a warmwater objective slightly cooler water would be released.

24. With $\bar{T}_{t,1}$ specified by the DP algorithm, the thermal simulation model is run for the first period using the initial temperature profile or state. The ports are regulated to achieve a release temperature $T_{r,1}$ as close as possible to the modified target $\bar{T}_{t,1}$. If the simulation model cannot produce a release temperature $T_{r,1}$ that achieves the modified target temperature $\bar{T}_{t,1}$, then a term $P(T_{r,1},\bar{T}_{t,1})$ is added to ΔD_1 as a penalty for missing the target temperature. For stage 1, there is actually no minimization needed, and the optimal value function is

$$F_1(D_1) = \left[\Delta D_1 + T(T_{r,1},\overline{T}_{t,1})\right] \tag{13}$$

where

$$\Delta D_1 = D_1 - D_0 = D_1 \tag{14}$$

25. To summarize, specification of discrete D_1 values yields ΔD_1 which in turn gives $\overline{T}_{t,i}$ (from Equation 12). The modified target is input to the simulation model and the ports are regulated to achieve that modified target as closely as possible. Any discrepancy is penalized in Equation 13. The temperature profile at the end of period 1 (or beginning of period 2) is $\underline{x}_1(D_1)$, which is also determined by the model and stored as a function of each discrete D_1; that is,

$$\underline{x}_1 = \underline{g}_1\left[\underline{x}_0,\underline{u}_1(\Delta D_1)\right] \tag{15}$$

where $u_1(\Delta D_1)$ is the port operational strategy selected based on a specified target modification ΔD_1. $\underline{g}_1$ is the stage transformation function to map the states from stage 0 to stage 1.

26. In stage 2, the optimal value function is defined by

$$F_2(D_2) = \min_{D_1}\left[\Delta D_2 + P(T_{r,2},\overline{T}_{t,2}) + F_1(D_1)\right] \tag{16}$$

where

$$\Delta D_2 = D_2 - D_1 \tag{17}$$

$$\overline{T}_{t,2} = T_{t,2} + (\Delta D_2)^{1/2} \tag{18}$$

and $T_{r,2}$ is obtained from the thermal simulation model. It may not be possible for $T_{r,2}$ to exactly equal $\overline{T}_{t,2}$ if the desired temperature is simply not available in the reservoir or if the outlet structure does not have sufficient flexibility to release the desired temperature. Notice that $F_2(D_2)$ can be written

$$F_2(D_2) = D_2 + \min_{D_1} \left[P(T_{r,2}, \overline{T}_{t,2}) + F_1(D_1) - D_1 \right] \tag{19}$$

where

$$F_1(D_1) - D_1 = P(T_{r,1}, \overline{T}_{t,1})^*$$

If it were possible to exactly achieve the modified targets $\overline{T}_{t,i}$ for $i = 1, 2, \ldots, N$, then

$$F_2(D_2) = D_2 \tag{20}$$

The minimizing operation in Equation 16 is simply a means of finding the operational strategy that achieves D_2 as closely as possible. The reason that D_2 is included in the right-hand side of Equation 19, even though it is not directly included in the minimization, will be shown subsequently.

27. As before, the simulation model determines $T_{r,2}$ as well as the resulting temperature state at the end of period 2 (or beginning of period 3)

$$\underline{x}_2 = \underline{g}_2\left[\underline{x}_1(D_1), \underline{u}_2(\Delta D_2)\right] \tag{21}$$

which is stored as a function of D_2 for the corresponding optimal D_1; that is, store $\underline{x}_2^*(D_2)$. The optimal deviation $\Delta D_2^*(D_2)$ as a function of D_2 is also stored.

28. It is important that the optima found in Equation 16 are unique for each discrete D_2 and that the resultant port operational strategies are also unique. Otherwise, there could be several possible end-of-period temperature profiles for the same release temperature target. Each set of profiles would have to be stored, quickly exhausting available computer storage during succeeding stages. The basic premise here is that it is usually not difficult to find a unique operational strategy for real systems once an operating target has been specified. There are generally many other explicit and implicit

objectives that govern the operation of the system. These include hydraulic, environmental, and even institutional factors, restrictions (constraints) which actually enhance the capability of this algorithm. This dynamic programming algorithm can regulate the daily targets to achieve an overall optimum for the entire operation.

29. For any stage i, the general formulation is

$$F_i(D_i) = \min_{D_{i-1}} \left[\Delta D_i + P(T_{r,i}, \overline{T}_{t,i}) + F_{i-1}(D_{i-1})\right] \tag{22}$$

where

$$\Delta D_i = D_i - D_{i-1} \tag{23}$$

$$\overline{T}_{t,i} = T_{t,i} + (\Delta D_i)^{1/2} \tag{24}$$

and $T_{r,i}$ is obtained from the model using the discrete temperature profile $\underline{x}^*_{i-1}(D_{i-1})$ stored from stage i - 1. The value of D_i represents the accumulated squared deviations of actual target temperature T_t from the DP-modified target temperature $\overline{T}_t$. By definition, $F_i(D_i)$ is the sum of accumulated squared deviations D_i plus minimum total penalties from stage 1 through stage i. As before, the optimal D_{i-1} value is found for each discrete D_i and the associated optimal ΔD_i (squared temperature deviation from $T_{t,i}$) for stage i is stored; that is, $\Delta D^*_i(D^*_i)$ is stored. The associated discrete temperature profiles resulting from these optimal values $\underline{x}^*_i(D^*_i)$ are also stored for use in the next stage. A flowchart for the computation process is given in Figure 6.

30. Recursive solution of Equation 22 will eventually yield the function $F_L(D_L)$ for each discrete value of D_L, where D_L is the total squared deviations between the modified target temperature and the actual target temperature. The function $F_L(D_L)$ is equal to D_L plus any additional penalties accumulated over periods 1, ..., L due to deviations between the specified modified target temperature and what can actually be achieved. A hypothetical plot is shown in Figure 7. The solid line assumes an ideal case where it is possible to find a

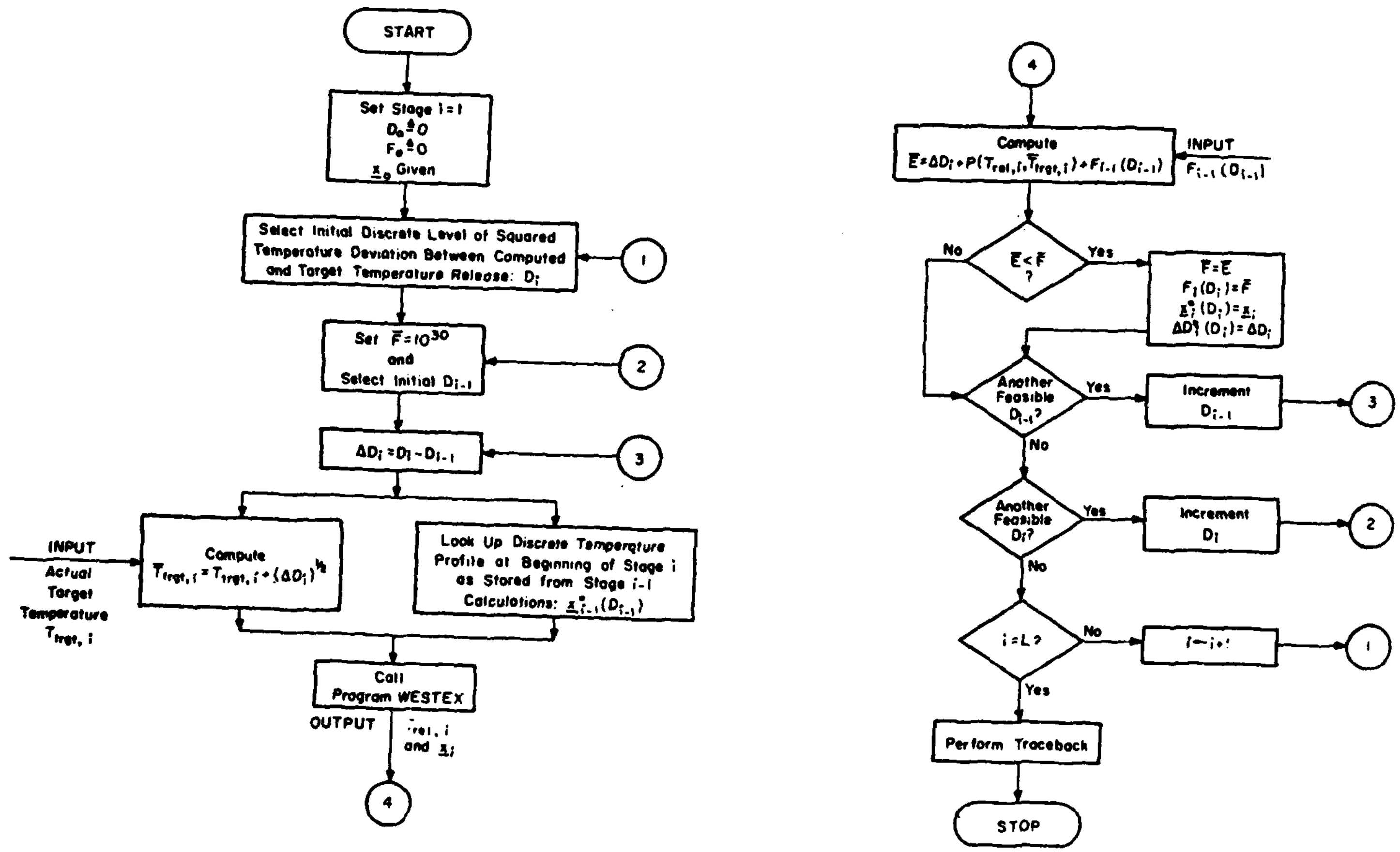


Figure 6. Flowchart for the objective-space dynamic programming procedure

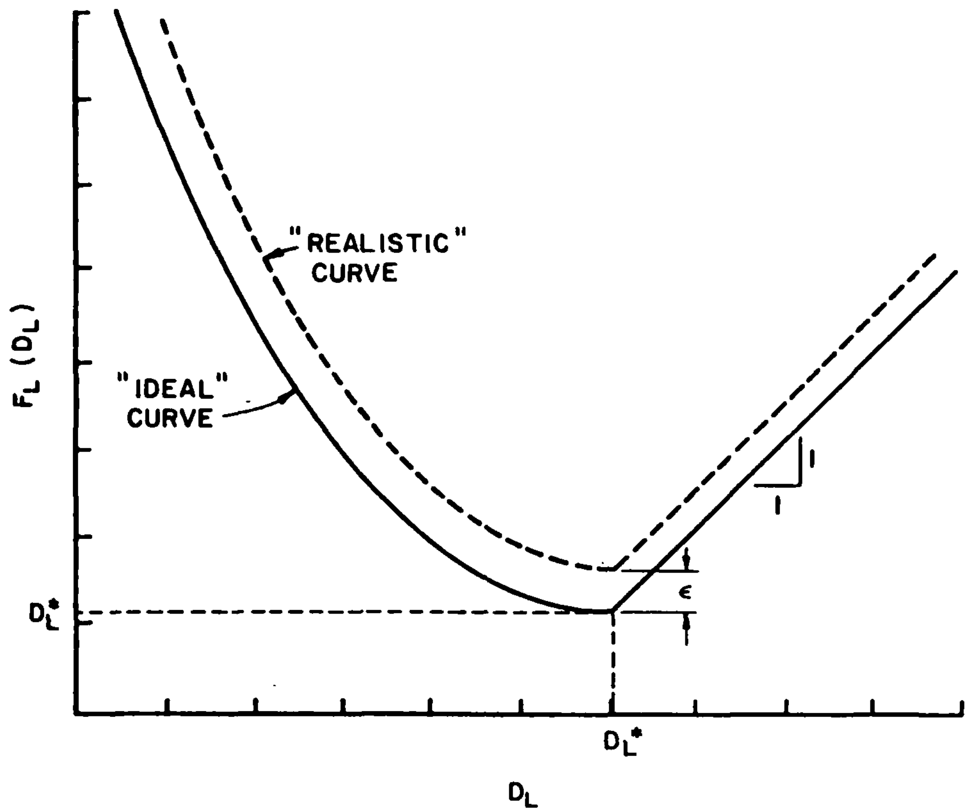


Figure 7. Theoretical relationship between $F_L(D_L)$ and D_L

feasible set of release temperatures without incurring any penalties. The minimal value is D_L^*. Notice that values less than D_L^* are not feasible and that the penalty terms therefore dominate $F_L(D_L)$. The function is linear for values of D_L greater than D_L^* since $F_L(D_L) = D_L$ in this region; that is, if D_L^* is feasible, it will generally be feasible to release warmer water--up to a point, of course.

31. A more realistic condition is shown by the dotted line in Figure 7. Here there is still some penalty, even at D_L^*. Thus,

$$F_L(D_L^*) = D_L^* + \varepsilon \tag{25}$$

If ε is within a desired order of accuracy, then this solution will suffice. Otherwise, the discretization interval used for D_i is probably too coarse, and the entire procedure should be repeated using a finer one. The bounds on D_i can be narrowed for this subsequent

optimization, based on results from the coarse discretization, in order to stabilize computational cost. Notice that if the ΔD_i values were not carried in Equation 23, there would not be an identifiable minimum to select: the plot would simply be a nonincreasing function. Also, if the minimum is a flat region, rather than a distinct minimum point, there are no unique solutions. An arbitrary point could be selected but it would have to be regarded as a poor solution. The only way to guarantee the best solution would be to carry all possible combinations of all possible nonunique solutions through the DP algorithm, which would likely be computationally impossible.

32. Once a specific D_L^* is selected, a traceback process is carried out to find the optimal modified target temperatures $\bar{T}_{t,i}^*$ for each stage. This is accomplished using the stored $\Delta D_i^*(D_i)$ values. The problem takes the form

Find $\Delta D_L^*(D_L^*)$
then

$$\bar{T}_{t,L}^* = T_{t,L} + (\Delta D_L^*)^{1/2} \tag{26}$$

$$D_{L-1}^* = D_L^* - \Delta D_L^* \tag{27}$$

Now find $\Delta D_{L-1}^*(D_{L-1}^*)$
and

$$\bar{T}_{t,L-1}^* = T_{t,L-1} + (\Delta D_{L-1}^*) \tag{28}$$

and so on. With the optimal modified target temperatures $\bar{T}_{t,i}$ determined for $i = 1, \ldots, L$, the simulation model can now be run with these targets to produce the optimal port operational strategies $\underline{u}_i^*$, $i = 1, \ldots, L$.

33. The reason for using a forward-looking DP procedure can now be seen more clearly. The forward approach begins at the first stage rather than the last. This is convenient because it assumes there is a specific temperature profile from which to start. The specific

temperature profiles resulting from the initial profile and the computed optimal target temperatures can be easily carried forward from stage to stage. The backward approach, on the other hand, would impose an immense computational burden. At each intermediate stage, there would be little guidance as to what starting temperature profile should be used since the initial stages would not yet have been evaluated. The purpose of the OSDP approach would be defeated since the intermediate stage problem would have to be solved for all possible discrete temperature profiles that could occur.

PART VI: COMPUTATIONAL EXPERIENCE

34. The OSDP approach was applied to the operation of a reservoir in northwestern Pennsylvania. A generalized one-dimensional dynamic programming code (CSU*DP) developed by Labadie and Shafer (1980) was linked with the WESTEX simulation model through a controlling executive program that maintained the integrity of both models. The combined WESTEX-CSU*DP model was used to develop an improved operational strategy for the case study reservoir for a 14-week period. A comparison between the squared deviations of computed release temperature from the downstream target temperature obtained by the normal myopic approach of the simulation model and those found by the OSDP approach is shown in Figure 8. A plot

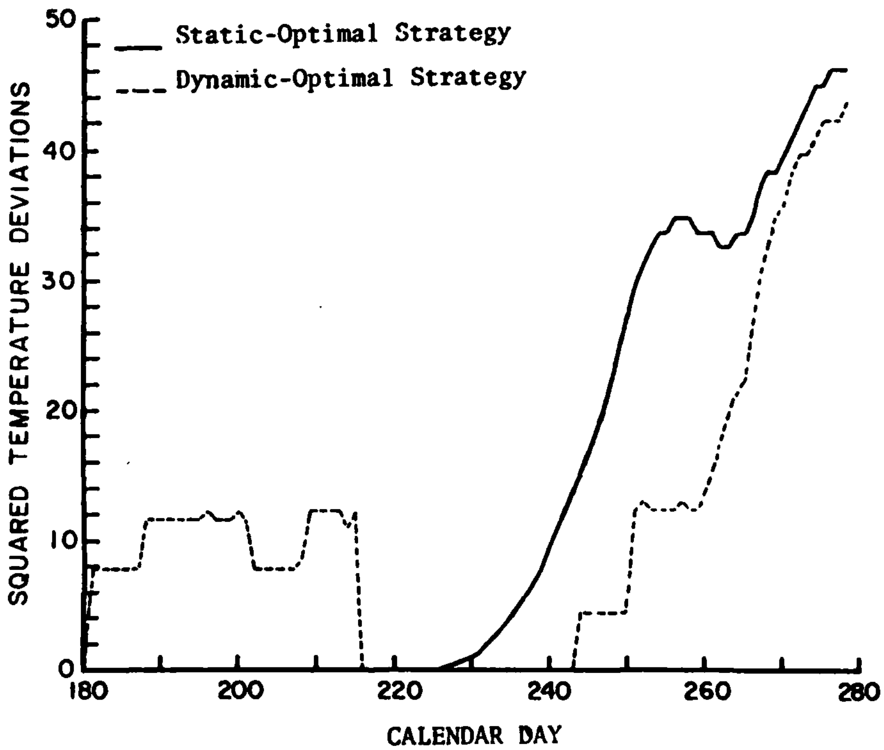


Figure 8. Squared deviations of predicted release temperature from downstream target temperature (°C)

of the values of $F_L(D_L)$ versus D_L obtained with the OSDP approach is shown in Figure 9. For this case, the penalty term used was of the form

$$P = (\bar{T}_{t,i} - T_{r,i})^2 + 2(\bar{T}_{t,i} - T_{r,i})(T_{t,i} - \bar{T}_{t,i}) \tag{29}$$

Note that Equation 29 is such that $F_L(D_L)$ actually represents the relationship of Equation 7; that is, the minimum sum of squared deviations of the predicted release temperature from the original target temperature.

$$F_L(D_L) = \min_{D_L} (T_{r,L} - T_{t,L})^2 \tag{30}$$

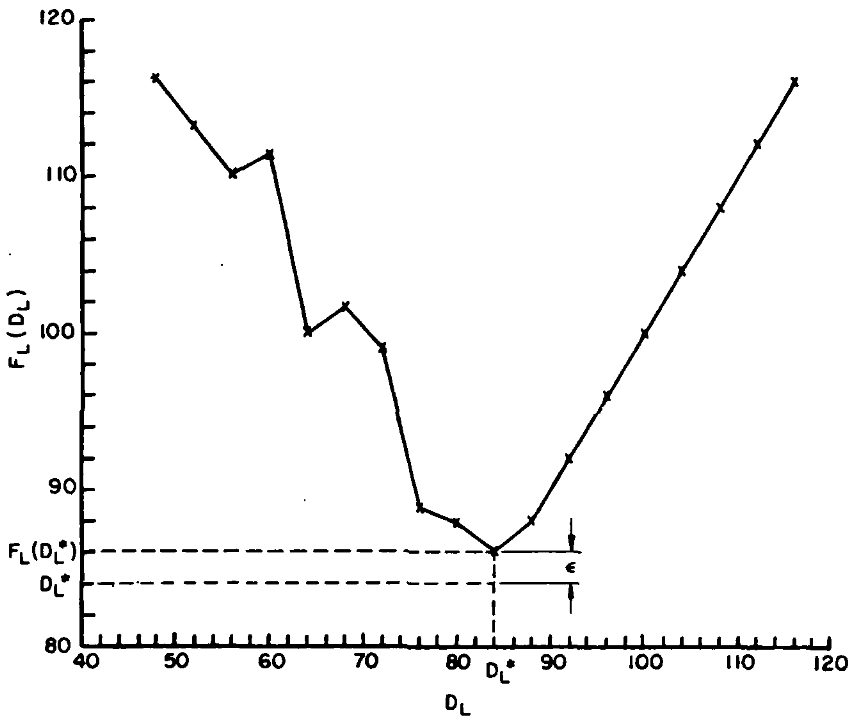


Figure 9. Computed relationship between $F_L(D_L)$ and D_L

35. The results shown in Figure 8 demonstrate the ability of the OSDP approach to find operational strategies that consider future conditions. The myopic (static-optimal) strategy incurs no deviations early in the operation period, but eventually the coldwater supply is depleted and large deviations occur toward the end of the period. The operational strategies determined by the combined WESTEX-CSU*DP model release warmer water from the upper level ports early in the period,

thereby incurring some deviations. However, the coldwater supply under this strategy is not so rapidly exhausted, and the deviations incurred toward the end of the operational period are smaller.

36. As discussed in the formulation of the objective-space technique, the uniqueness of the D_L^* values must be determined. The CSU*DP code contains logic to "break ties" if multiple values of the minimizing variable yield identical values of the optimal return function; that is, if there are multiple optimum trajectories of the state variable. The user can specify that either the first or the last tie value be selected. The results shown in Figure 8 were obtained with first-tie selection. The problem was then solved using last-tie selection and the results were identical. Therefore, a unique optimal state trajectory exists, and the uniqueness of the D_i^* values for this problem is strictly guaranteed for the test case.

37. Although the plot of $F_L(D_L)$ versus D_L shown in Figure 9 exhibits the theoretical relationship of Figure 7, the relationship for $D_L < D_L^*$ is not monotonic as anticipated. Conceptually, as the infeasibility of the D_L values increases ($D_L << D_L^*$), the magnitude of the penalty incurred should increase accordingly. Why it did not could not be exactly determined for this analysis. It is felt, however, that the non-monotonic relationship was related to the penalty function used (Equation 29) and that a different penalty function or objective function should produce the expected monotonic relationship.

38. To evaluate operational strategies for a different objective function, another 14-week period was analyzed by the combined WESTEX-CSU*DP model with the objective of minimizing the maximum deviation that would occur during the period. A comparison of the deviations of computed release temperature from the downstream target temperature obtained by the myopic approach and those found using the OSDP approach is shown in Figure 10. Minimizing the maximum deviation yields an operational strategy that incurs deviations throughout the period; however, the maximum deviation is approximately 50 percent greater for the myopic strategy.

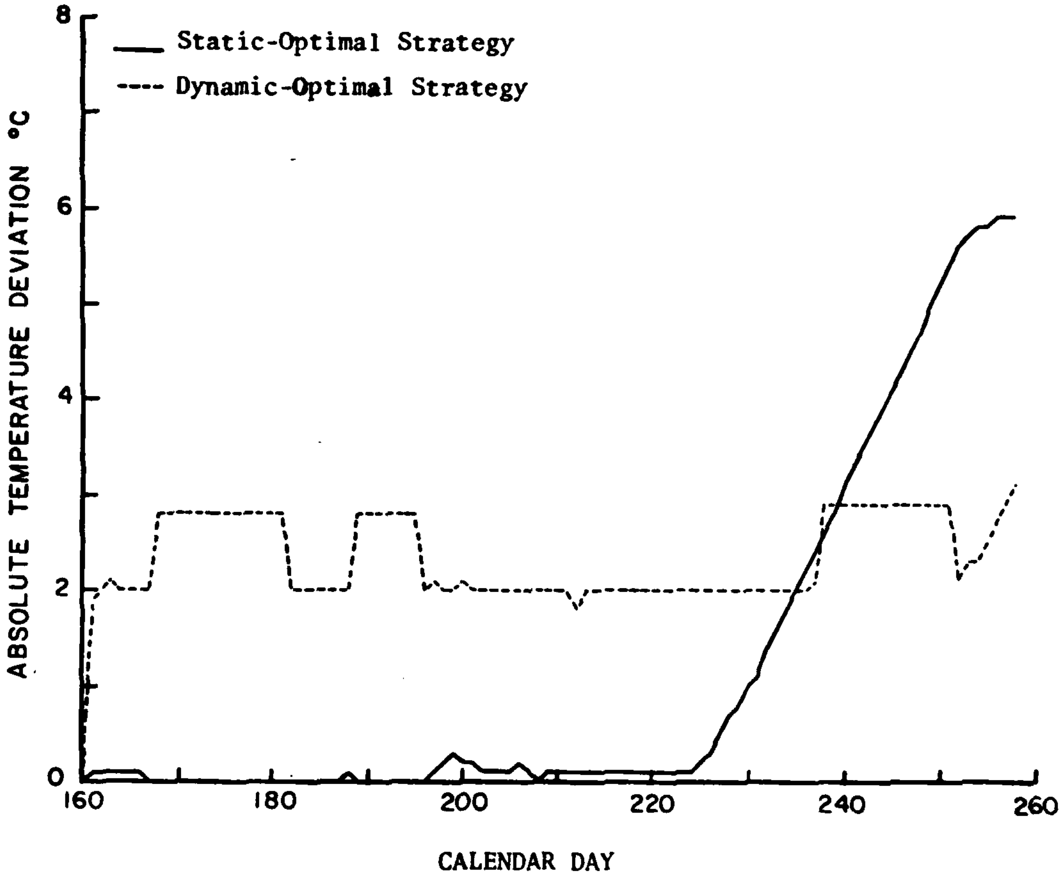


Figure 10. Deviations of release temperature from target temperature (°C) for a minmax objective function

PART VII: CONCLUSIONS AND EXTENSIONS

39. Combining a thermal simulation model with a dynamic programming optimization algorithm and anticipation of future hydrologic and meteorologic conditions have been demonstrated to yield improved operational strategies for selective withdrawal structures. The objective-space dynamic programming approach reduces a large multidimensional problem to its equivalent in one dimension and therefore eliminates many computational difficulties. Also, the resulting one-dimensional DP problem is much more amenable to a stochastic DP approach which recognizes that forecasted information is uncertain and which could be used to evaluate the effects of forecast lead time upon accuracy.

40. Because of the nature of the objective-space approach, the combined WESTEX-CSU*DP model could be formulated such that the basic structures of both the WESTEX simulation model and the CSU*DP optimizing model need not be altered. The combined WESTEX-CSU*DP model can be used to determine operational strategies for a wide range of temperature objectives, such as minimizing the sum of absolute deviations or minimizing the sum of weighted deviations. The approach is not limited to a particular reservoir simulation model. Theoretically, any reservoir thermal simulation model may be used if it employs a multilevel selective withdrawal algorithm to choose the port operations required to meet daily objectives. Additionally, while the problem presented herein focused on the need to save cold water in a reservoir, the approach could just as easily have been used for maintaining a warmwater release.

41. Finally, the method could be extended to evaluate other water quality parameters in addition to release temperature. The objectives for these other parameters could be treated as constraints in the one-dimensional dynamic programming problem; for example, minimizing the sum of squared temperature deviations subject to maintaining the dissolved oxygen content of the release above some specified limit. Alternately, other water quality parameter objectives could be handled directly by defining an additional state variable for each parameter and

solving the problem over multidimensional objective space.

42. It is believed that there are many possibilities for application of objective-space dynamic programming within the field of water resources systems analysis, in particular, problems which have defied solution because of their dimensionality.

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Optimal control of reservoir discharge quality through selective withdrawal : Hydraulic laboratory investigation / by Darrell G. Fontane, John W. Labadie (Department of Civil Engineering, Colorado State University), Bruce Loftis (Hydraulics Laboratory, U.S. Army Engineer Waterways Experiment Station). -- Vicksburg, Miss. : The Station ; Springfield, Va. : available from NTIS, 1982.
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